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Monterey, California

PRIMES
THE FIRST TWO THOUSAND FOUR HUNDRED PRIME NUMBERS

GILBERT FORD KINNEY

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PRIMES

The First
Two Thousand Four Hundred
Prime Numbers

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PRIMES

The First Two Thousand
Four Hundred Prime Numbers

PREFACE

These simple and mathematically elegant but practically useless prime number listings could have an appeal for afficionados of elementary number theory. They were prepared using a computer adaptation of the Sieve of Aratosthenes of Alexandria and the computations made on a small personal computer with an 8-bit microprocesssor, a 64K random access memory, and a 2-megahertz clock. Computing time for checking 21.380 integers and identifying the included 2400 prime numbers was about thirty minutes. This computational effort is quite modest compared to others such as two which are reported to have examined the first ten million integers. But the mere 2400 primes reported here, plus related items such as the number of prime twins and the integer gap between successive primes, are presented in tangible form.

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prime, *adj.* [fr. L. *primus* first]

5. *Math.* Divisible by no number except itself or unity; as, 7 is a *prime* number.

6. *n. Arith.* A prime number.

Introduction

Prime numbers, also termed primes, are divisible only by themselves and unity. Prime numbers often have been thought to have mystical significance. For example the prime number 7 is assigned special signifance in the first chapter of the first book of the Bible. The prime number 7 is often considered lucky, and the prime number 13 unlucky.

Mathematical interest in the prime numbers dates from about 300 B.C. when the famous Greek geometer Euclid of Alexandria showed that there is no limit to the number of primes. About a century later the Greek astronomer Eratosthenes, also of Alexandria, provided a method for identifying prime numbers and determining the number of primes not greater than some specified integer. (Eratosthenes also determined the earth's circumference by measuring the distance along the meridian from Alexanderia to the equator.)

The study of prime numbers is a part of number theory (sometimes called "the higher arithmetic"). Modern study of number theory began with the French lawyer and mathematician Pierre de Fermat in the seventeenth century. Fermat himself published very little and our knowledge of his work comes from his correspondence with mathematically inclined friends. Fermat developed a method for factorization of large integers, incidently permitting identification of those which are primes. An antedote about him is that he once was asked was the integer 100,895,598,169 a prime number. He replied that it was not, and that it was the product of two prime numbers 898,423 and 112,303. Fermat's famous last theorem, which is still unproven, is that there are no integral values for x, y, and z in the equation $x^n + y^n = z^n$ where n is an integer larger than 2.

Later the prolific French scientist Leonhard Euler devised an additional factorization method. Then early in the nineteenth century the mathematicians Adrien Marie Legendre and Karl Frederick Gauss conjected a "Prime-number Theorem" providing formulas for determining which very large integers are also prime numbers, and the number of primes not greater than a specified large integer. This "Prime-number Theorem" conjecture is the basis for a subsequent section. It was some years later that the French mathematician J. Hadanard and the Belgian mathematician C. J. de la Vallee-Poussin acting independently proved that this theorem is correct.

The largest integer that until recently had been verified as being a prime number was found in 1876 by the French mathematician Lucas. This prime number has 39 digits and is reported here as a show of erudition:

170,141,183,460,469,231,731,687,303,715,884,105,727 .

Since then the range of known prime numbers has been greatly extended by use of modern computers. Currently, the largest known prime, indentified in 1987, is the 65,050 digit integer that would require twenty or more closely typed pages for its presentation. There is no reason to believe that even larger primes await to be discovered.

Recently a list of 850 very large prime numbers each with one thousand digits or more and which can be represented algebraically in a single typed line, has been published. These of course constitute a small fraction of the total number of primes within this range. Thus the prime theorem conjecture, mentioned subsequently, indicates that there are something in the order of 4.3 x 10^996 prime numbers with no more than one thousand digits, many of which would require at least one third of a typed page for presentation.

1. The First 2400 Prime Numbers

The first 21,380 consecutive integers that are considered in this program include two thousand four hundred integers that also are prime numbers. These prime numbers are listed here in the following six pages in sets of 10, 50, and 400.

The First 2400 Prime Numbers

(in sets of 400)

1	2	3	5	7	11	13	17	19	23
29	31	37	41	43	47	53	59	61	67
71	73	79	83	89	97	101	103	107	109
113	127	131	137	139	149	151	157	163	167
173	179	181	191	193	197	199	211	223	227
229	233	239	241	251	257	263	269	271	277
281	283	293	307	311	313	317	331	337	347
349	353	359	367	373	379	383	389	397	401
409	419	421	431	433	439	443	449	457	461
463	467	479	487	491	499	503	509	521	523
541	547	557	563	569	571	577	587	593	599
601	607	613	617	619	631	641	643	647	653
659	661	673	677	683	691	701	709	719	727
733	739	743	751	757	761	769	773	787	797
809	811	821	823	827	829	839	853	857	859
863	877	881	883	887	907	911	919	929	937
941	947	953	967	971	977	983	991	997	1009
1013	1019	1021	1031	1033	1039	1049	1051	1061	1063
1069	1087	1091	1093	1097	1103	1109	1117	1123	1129
1151	1153	1163	1171	1181	1187	1193	1201	1213	1217
1223	1229	1231	1237	1249	1259	1277	1279	1283	1289
1291	1297	1301	1303	1307	1319	1321	1327	1361	1367
1373	1381	1399	1409	1423	1427	1429	1433	1439	1447
1451	1453	1459	1471	1481	1483	1487	1489	1493	1499
1511	1523	1531	1543	1549	1553	1559	1567	1571	1579
1583	1597	1601	1607	1609	1613	1619	1621	1627	1637
1657	1663	1667	1669	1693	1697	1699	1709	1721	1723
1733	1741	1747	1753	1759	1777	1783	1787	1789	1801
1811	1823	1831	1847	1861	1867	1871	1873	1877	1879
1889	1901	1907	1913	1931	1933	1949	1951	1973	1979
1987	1993	1997	1999	2003	2011	2017	2027	2029	2039
2053	2063	2069	2081	2083	2087	2089	2099	2111	2113
2129	2131	2137	2141	2143	2153	2161	2179	2203	2207
2213	2221	2237	2239	2243	2251	2267	2269	2273	2281
2287	2293	2297	2309	2311	2333	2339	2341	2347	2351
2357	2371	2377	2381	2383	2389	2393	2399	2411	2417
2423	2437	2441	2447	2459	2467	2473	2477	2503	2521
2531	2539	2543	2549	2551	2557	2579	2591	2593	2609
2617	2621	2633	2647	2657	2659	2663	2671	2677	2683
2687	2689	2693	2699	2707	2711	2713	2719	2729	2731

The First 2400 Prime Numbers

(in sets of 400)

2741	2749	2753	2767	2777	2789	2791	2797	2801	2803
2819	2833	2837	2843	2851	2857	2861	2879	2887	2897
2903	2909	2917	2927	2939	2953	2957	2963	2969	2971
2999	3001	3011	3019	3023	3037	3041	3049	3061	3067
3079	3083	3089	3109	3119	3121	3137	3163	3167	3169

3181	3187	3191	3203	3209	3217	3221	3229	3251	3253
3257	3259	3271	3299	3301	3307	3313	3319	3323	3329
3331	3343	3347	3359	3361	3371	3373	3389	3391	3407
3413	3433	3449	3457	3461	3463	3467	3469	3491	3499
3511	3517	3527	3529	3533	3539	3541	3547	3557	3559

3571	3581	3583	3593	3607	3613	3617	3623	3631	3637
3643	3659	3671	3673	3677	3691	3697	3701	3709	3719
3727	3733	3739	3761	3767	3769	3779	3793	3797	3803
3821	3823	3833	3847	3851	3853	3863	3877	3881	3889
3907	3911	3917	3919	3923	3929	3931	3943	3947	3967

3989	4001	4003	4007	4013	4019	4021	4027	4049	4051
4057	4073	4079	4091	4093	4099	4111	4127	4129	4133
4139	4153	4157	4159	4177	4201	4211	4217	4219	4229
4231	4241	4243	4253	4259	4261	4271	4273	4283	4289
4297	4327	4337	4339	4349	4357	4363	4373	4391	4397

4409	4421	4423	4441	4447	4451	4457	4463	4481	4483
4493	4507	4513	4517	4519	4523	4547	4549	4561	4567
4583	4591	4597	4603	4621	4637	4639	4643	4649	4651
4657	4663	4673	4679	4691	4703	4721	4723	4729	4733
4751	4759	4783	4787	4789	4793	4799	4801	4813	4817

4831	4861	4871	4877	4889	4903	4909	4919	4931	4933
4937	4943	4951	4957	4967	4969	4973	4987	4993	4999
5003	5009	5011	5021	5023	5039	5051	5059	5077	5081
5087	5099	5101	5107	5113	5119	5147	5153	5167	5171
5179	5189	5197	5209	5227	5231	5233	5237	5261	5273

5279	5281	5297	5303	5309	5323	5333	5347	5351	5381
5387	5393	5399	5407	5413	5417	5419	5431	5437	5441
5443	5449	5471	5477	5479	5483	5501	5503	5507	5519
5521	5527	5531	5557	5563	5569	5573	5581	5591	5623
5639	5641	5647	5651	5653	5657	5659	5669	5683	5689

5693	5701	5711	5717	5737	5741	5743	5749	5779	5783
5791	5801	5807	5813	5821	5827	5839	5843	5849	5851
5857	5861	5867	5869	5879	5881	5897	5903	5923	5927
5939	5953	5981	5987	6007	6011	6029	6037	6043	6047
6053	6067	6073	6079	6089	6091	6101	6113	6121	6131

The First 2400 Prime Numbers

(in sets of 400)

6133	6143	6151	6163	6173	6197	6199	6203	6211	6217
6221	6229	6247	6257	6263	6269	6271	6277	6287	6299
6301	6311	6317	6323	6329	6337	6343	6353	6359	6361
6367	6373	6379	6389	6397	6421	6427	6449	6451	6469
6473	6481	6491	6521	6529	6547	6551	6553	6563	6569

6571	6577	6581	6599	6607	6619	6637	6653	6659	6661
6673	6679	6689	6691	6701	6703	6709	6719	6733	6737
6761	6763	6779	6781	6791	6793	6803	6823	6827	6829
6833	6841	6857	6863	6869	6871	6883	6899	6907	6911
6917	6947	6949	6959	6961	6967	6971	6977	6983	6991

6997	7001	7013	7019	7027	7039	7043	7057	7069	7079
7103	7109	7121	7127	7129	7151	7159	7177	7187	7193
7207	7211	7213	7219	7229	7237	7243	7247	7253	7283
7297	7307	7309	7321	7331	7333	7349	7351	7369	7393
7411	7417	7433	7451	7457	7459	7477	7481	7487	7489

7499	7507	7517	7523	7529	7537	7541	7547	7549	7559
7561	7573	7577	7583	7589	7591	7603	7607	7621	7639
7643	7649	7669	7673	7681	7687	7691	7699	7703	7717
7723	7727	7741	7753	7757	7759	7789	7793	7817	7823
7829	7841	7853	7867	7873	7877	7879	7883	7901	7907

7919	7927	7933	7937	7949	7951	7963	7993	8009	8011
8017	8039	8053	8059	8069	8081	8087	8089	8093	8101
8111	8117	8123	8147	8161	8167	8171	8179	8191	8209
8219	8221	8231	8233	8237	8243	8263	8269	8273	8287
8291	8293	8297	8311	8317	8329	8353	8363	8369	8377

8387	8389	8419	8423	8429	8431	8443	8447	8461	8467
8501	8513	8521	8527	8537	8539	8543	8563	8573	8581
8597	8599	8609	8623	8627	8629	8641	8647	8663	8669
8677	8681	8689	8693	8699	8707	8713	8719	8731	8737
8741	8747	8753	8761	8779	8783	8803	8807	8819	8821

8831	8837	8839	8849	8861	8863	8867	8887	8893	8923
8929	8933	8941	8951	8963	8969	8971	8999	9001	9007
9011	9013	9029	9041	9043	9049	9059	9067	9091	9103
9109	9127	9133	9137	9151	9157	9161	9173	9181	9187
9199	9203	9209	9221	9227	9239	9241	9257	9277	9281

9283	9293	9311	9319	9323	9337	9341	9343	9349	9371
9377	9391	9397	9403	9413	9419	9421	9431	9433	9437
9439	9461	9463	9467	9473	9479	9491	9497	9511	9521
9533	9539	9547	9551	9587	9601	9613	9619	9623	9629
9631	9643	9649	9661	9677	9679	9689	9697	9719	9721

The First 2400 Prime Numbers

(in sets of 400)

9733	9739	9743	9749	9767	9769	9781	9787	9791	9803
9811	9817	9829	9833	9839	9851	9857	9859	9871	9883
9887	9901	9907	9923	9929	9931	9941	9949	9967	9973
10007	10009	10037	10039	10061	10067	10069	10079	10091	10093
10099	10103	10111	10133	10139	10141	10151	10159	10163	10169
10177	10181	10193	10211	10223	10243	10247	10253	10259	10267
10271	10273	10289	10301	10303	10313	10321	10331	10333	10337
10343	10357	10369	10391	10399	10427	10429	10433	10453	10457
10459	10463	10477	10487	10499	10501	10513	10529	10531	10559
10567	10589	10597	10601	10607	10613	10627	10631	10639	10651
10657	10663	10667	10687	10691	10709	10711	10723	10729	10733
10739	10753	10771	10781	10789	10799	10831	10837	10847	10853
10859	10861	10867	10883	10889	10891	10903	10909	10937	10939
10949	10957	10973	10979	10987	10993	11003	11027	11047	11057
11059	11069	11071	11083	11087	11093	11113	11117	11119	11131
11149	11159	11161	11171	11173	11177	11197	11213	11239	11243
11251	11257	11261	11273	11279	11287	11299	11311	11317	11321
11329	11351	11353	11369	11383	11393	11399	11411	11423	11437
11443	11447	11467	11471	11483	11489	11491	11497	11503	11519
11527	11549	11551	11579	11587	11593	11597	11617	11621	11633
11657	11677	11681	11689	11699	11701	11717	11719	11731	11743
11777	11779	11783	11789	11801	11807	11813	11821	11827	11831
11833	11839	11863	11867	11887	11897	11903	11909	11923	11927
11933	11939	11941	11953	11959	11969	11971	11981	11987	12007
12011	12037	12041	12043	12049	12071	12073	12097	12101	12107
12109	12113	12119	12143	12149	12157	12161	12163	12197	12203
12211	12227	12239	12241	12251	12253	12263	12269	12277	12281
12289	12301	12323	12329	12343	12347	12373	12377	12379	12391
12401	12409	12413	12421	12433	12437	12451	12457	12473	12479
12487	12491	12497	12503	12511	12517	12527	12539	12541	12547
12553	12569	12577	12583	12589	12601	12611	12613	12619	12637
12641	12647	12653	12659	12671	12689	12697	12703	12713	12721
12739	12743	12757	12763	12781	12791	12799	12809	12821	12823
12829	12841	12853	12889	12893	12899	12907	12911	12917	12919
12923	12941	12953	12959	12967	12973	12979	12983	13001	13003
13007	13009	13033	13037	13043	13049	13063	13093	13099	13103
13109	13121	13127	13147	13151	13159	13163	13171	13177	13183
13187	13217	13219	13229	13241	13249	13259	13267	13291	13297
13309	13313	13327	13331	13337	13339	13367	13381	13397	13399
13411	13417	13421	13441	13451	13457	13463	13469	13477	13487

The First 2400 Prime Numbers

(in sets of 400)

13499	13513	13523	13537	13553	13567	13577	13591	13597	13613
13619	13627	13633	13649	13669	13679	13681	13687	13691	13693
13697	13709	13711	13721	13723	13729	13751	13757	13759	13763
13781	13789	13799	13807	13829	13831	13841	13859	13873	13877
13879	13883	13901	13903	13907	13913	13921	13931	13933	13963
13967	13997	13999	14009	14011	14029	14033	14051	14057	14071
14081	14083	14087	14107	14143	14149	14153	14159	14173	14177
14197	14207	14221	14243	14249	14251	14281	14293	14303	14321
14323	14327	14341	14347	14369	14387	14389	14401	14407	14411
14419	14423	14431	14437	14447	14449	14461	14479	14489	14503
14519	14533	14537	14543	14549	14551	14557	14561	14563	14591
14593	14621	14627	14629	14633	14639	14653	14657	14669	14683
14699	14713	14717	14723	14731	14737	14741	14747	14753	14759
14767	14771	14779	14783	14797	14813	14821	14827	14831	14843
14851	14867	14869	14879	14887	14891	14897	14923	14929	14939
14947	14951	14957	14969	14983	15013	15017	15031	15053	15061
15073	15077	15083	15091	15101	15107	15121	15131	15137	15139
15149	15161	15173	15187	15193	15199	15217	15227	15233	15241
15259	15263	15269	15271	15277	15287	15289	15299	15307	15313
15319	15329	15331	15349	15359	15361	15373	15377	15383	15391
15401	15413	15427	15439	15443	15451	15461	15467	15473	15493
15497	15511	15527	15541	15551	15559	15569	15581	15583	15601
15607	15619	15629	15641	15643	15647	15649	15661	15667	15671
15679	15683	15727	15731	15733	15737	15739	15749	15761	15767
15773	15787	15791	15797	15803	15809	15817	15823	15859	15877
15881	15887	15889	15901	15907	15913	15919	15923	15937	15959
15971	15973	15991	16001	16007	16033	16057	16061	16063	16067
16069	16073	16087	16091	16097	16103	16111	16127	16139	16141
16183	16187	16189	16193	16217	16223	16229	16231	16249	16253
16267	16273	16301	16319	16333	16339	16349	16361	16363	16369
16381	16411	16417	16421	16427	16433	16447	16451	16453	16477
16481	16487	16493	16519	16529	16547	16553	16561	16567	16573
16603	16607	16619	16631	16633	16649	16651	16657	16661	16673
16691	16693	16699	16703	16729	16741	16747	16759	16763	16787
16811	16823	16829	16831	16843	16871	16879	16883	16889	16901
16903	16921	16927	16931	16937	16943	16963	16979	16981	16987
16993	17011	17021	17027	17029	17033	17041	17047	17053	17077
17093	17099	17107	17117	17123	17137	17159	17167	17183	17189
17191	17203	17207	17209	17231	17239	17257	17291	17293	17299
17317	17321	17327	17333	17341	17351	17359	17377	17383	17387

The First 2400 Prime Numbers

(in sets of 400)

17389	17393	17401	17417	17419	17431	17443	17449	17467	17471
17477	17483	17489	17491	17497	17509	17519	17539	17551	17569
17573	17579	17581	17597	17599	17609	17623	17627	17657	17659
17669	17681	17683	17707	17713	17729	17737	17747	17749	17761
17783	17789	17791	17807	17827	17837	17839	17851	17863	17881

17891	17903	17909	17911	17921	17923	17929	17939	17957	17959
17971	17977	17981	17987	17989	18013	18041	18043	18047	18049
18059	18061	18077	18089	18097	18119	18121	18127	18131	18133
18143	18149	18169	18181	18191	18199	18211	18217	18223	18229
18233	18251	18253	18257	18269	18287	18289	18301	18307	18311

18313	18329	18341	18353	18367	18371	18379	18397	18401	18413
18427	18433	18439	18443	18451	18457	18461	18481	18493	18503
18517	18521	18523	18539	18541	18553	18583	18587	18593	18617
18637	18661	18671	18679	18691	18701	18713	18719	18731	18743
18749	18757	18773	18787	18793	18797	18803	18839	18859	18869

18899	18911	18913	18917	18919	18947	18959	18973	18979	19001
19009	19013	19031	19037	19051	19069	19073	19079	19081	19087
19121	19139	19141	19157	19163	19181	19183	19207	19211	19213
19219	19231	19237	19249	19259	19267	19273	19289	19301	19309
19319	19333	19373	19379	19381	19387	19391	19403	19417	19421

19423	19427	19429	19433	19441	19447	19457	19463	19469	19471
19477	19483	19489	19501	19507	19531	19541	19543	19553	19559
19571	19577	19583	19597	19603	19609	19661	19681	19687	19697
19699	19709	19717	19727	19739	19751	19753	19759	19763	19777
19793	19801	19813	19819	19841	19843	19853	19861	19867	19889

19891	19913	19919	19927	19937	19949	19961	19963	19973	19979
19991	19993	19997	20011	20021	20023	20029	20047	20051	20063
20071	20089	20101	20107	20113	20117	20123	20129	20143	20147
20149	20161	20173	20177	20183	20201	20219	20231	20233	20249
20261	20269	20287	20297	20323	20327	20333	20341	20347	20353

20357	20359	20369	20389	20393	20399	20407	20411	20431	20441
20443	20477	20479	20483	20507	20509	20521	20533	20543	20549
20551	20563	20593	20599	20611	20627	20639	20641	20663	20681
20693	20707	20717	20719	20731	20743	20747	20749	20753	20759
20771	20773	20789	20807	20809	20849	20857	20873	20879	20887

20897	20899	20903	20921	20929	20939	20947	20959	20963	20981
20983	21001	21011	21013	21017	21019	21023	21031	21059	21061
21067	21089	21101	21107	21121	21139	21143	21149	21157	21163
21169	21179	21187	21191	21193	21211	21221	21227	21247	21269
21277	21283	21313	21317	21319	21323	21341	21347	21377	21379

2. Prime-Number Twins

(identified by the integer

between two successive primes)

A striking feature of the sequence of prime numbers is that many of them occur in pairs separated by a single integer. These pairs constitute prime-number twins. The first pair of such prime-number twins are the primes 3 and 5. Other examples include 11 and 13, 29 and 31, and 2687 and 2689. The integer separating these primes is necessarily an even number and the mean of the two prime numbers, and for the twins above this separating integer is 4, 12, 30, or 2688.

The accompanying table lists the separating integer for all pairs of twins occurring within the first 2400 prime numbers. Thus the non-prime integer 19752 identifies the pair of prime number-twins 19751 and 19753.

There are no prime-number "triplets" for integers larger than 5. Here the three successive odd-numbered integers always include

one that is divisible by 2 or by 5. Primes not greater than 5 are all special cases. Thus the integer 1 often is not considered to be a prime number, the prime number 2 is the only even-numbered prime, the prime number 3 is the only prime divisible by three, and 5 the only prime divisible by five.

This table for prime-number twins lists the separating integer in the first 368 pairs of twins for the first 2400 prime numbers (or the first 21,380 integers). These twinned primes constitute about 30% of the total number, and about 3 1/5 % of the number of integers. Both the number ot primes and of prime-number twins increase with increasing numbers of integers, but at a somewhat irregular rate. For example there are 169 primes and 35 pairs of twins in the first set of 1000 integers, but only 104 primes and 15 pairs of twins in the twentieth set of 1000 integers.

Prime-number Twins

(the integer between two successive primes)

4	6	12	18	30	42	60	72	102	108
138	150	180	192	198	228	240	270	282	312
348	420	432	462	522	570	600	618	642	660
810	822	828	858	882	1020	1032	1050	1062	1092
1152	1230	1278	1290	1302	1320	1428	1452	1482	1488
1608	1620	1668	1698	1722	1788	1872	1878	1932	1950
1998	2028	2082	2088	2112	2130	2142	2238	2268	2310
2340	2382	2550	2592	2658	2688	2712	2730	2790	2802
2970	3000	3120	3168	3252	3258	3300	3330	3360	3372
3390	3462	3468	3528	3540	3558	3582	3672	3768	3822
3852	3918	3930	4002	4020	4050	4092	4128	4158	4218
4230	4242	4260	4272	4338	4422	4482	4518	4548	4638
4650	4722	4788	4800	4932	4968	5010	5022	5100	5232
5280	5418	5442	5478	5502	5520	5640	5652	5658	5742
5850	5868	5880	6090	6132	6198	6270	6300	6360	6450
6552	6570	6660	6690	6702	6762	6780	6792	6828	6870
6948	6960	7128	7212	7308	7332	7350	7458	7488	7548
7560	7590	7758	7878	7950	8010	8088	8220	8232	8292
8388	8430	8538	8598	8628	8820	8838	8862	8970	9000
9012	9042	9240	9282	9342	9420	9432	9438	9462	9630
9678	9720	9768	9858	9930	10008	10038	10068	10092	10140
10272	10302	10332	10428	10458	10500	10530	10710	10860	10890
10938	11058	11070	11118	11160	11172	11352	11490	11550	11700
11718	11778	11832	11940	11970	12042	12072	12108	12162	12240
12252	12378	12540	12612	12822	12918	13002	13008	13218	13338
13398	13680	13692	13710	13722	13758	13830	13878	13902	13932
13998	14010	14082	14250	14322	14388	14448	14550	14562	14592
14628	14868	15138	15270	15288	15330	15360	15582	15642	15648
15732	15738	15888	15972	16062	16068	16140	16188	16230	16362
16452	16632	16650	16692	16830	16902	16980	17028	17190	17208
17292	17388	17418	17490	17580	17598	17658	17682	17748	17790
17838	17910	17922	17958	17988	18042	18048	18060	18120	18132
18252	18288	18312	18522	18540	18912	18918	19080	19140	19182
19212	19380	19422	19428	19470	19542	19698	19752	19842	19890
19962	19992	20022	20148	20232	20358	20442	20478	20508	20550
20640	20718	20748	20772	20808	20898	20982	21012	21018	21060
21192	21318	21378							

Number of Primes and Prime-Number Twins

(for sets of integers)

Ranges of the Integers			Cumulative Totals		
Range	Primes	Twins	Integer	Primes	Twins
1 - 1,000	169	35	1,000	169	35
1,001 - 2,000	135	26	2,000	304	61
2,001 - 3,000	127	20	3,000	431	81
3,001 - 4,000	120	23	4,000	551	103
4,001 - 5,000	119	23	5,000	670	126
5,001 - 6,000	114	17	6,000	784	143
6,001 - 7,000	117	19	7,000	901	162
7,001 - 8,000	107	13	8,000	1008	175
8,001 - 9,000	110	14	9,000	1118	189
9,001 - 10,000	112	16	10,000	1230	205
10,001 - 11,000	106	16	11,000	1336	221
11,001 - 12,000	103	14	12,000	1439	235
12,001 - 13,000	109	11	13,000	1548	246
13,001 - 14,000	105	15	14,000	1653	261
14,001 - 15,000	102	11	15,000	1755	272
15,001 - 16,000	108	12	16,000	1863	284
16,001 - 17,000	98	13	17,000	1961	297
17,001 - 18,000	104	18	18,000	2065	315
18,001 - 19,000	94	12	19,000	2159	327
19,001 - 20,000	104	15	20,000	2263	342
20,001 - 21,000	98	15	21,000	2361	357

3. The Integer Gap

Complexity of the sequence of prime numbers leads to many and varied integer gaps between successive primes. In general these gaps become larger as the successive prime numbers become larger, for the increasing number of separating integers provides more possible divisors. It has been shown that this integer gap increases without limit.

Maximum integer gaps between successive prime numbers are tabulated here for the primes occuring within the sequence of integers up to 21,380 and are presented for steps of one hundred successive primes. Also shown is the maximum gap that occurs for successive groups of one hundred primes for this range of integers.

The integer gap is always an odd number of integers. Although the gap increases with increasing size of the primes, the rate of increase is somewhat irregular. This is shown particularly in the table for the gaps within successive groups of one hundred primes.

Maximum Gaps Between Successive Primes

(for groups of primes)

Total primes	Total integers	Max gap	Bracketing primes	Range of primes	Max gap	Bracketing primes
100	523	13	113- 127	1- 100	13	113- 127
200	1217	21	1129- 1151	101- 200	21	1129- 1151
300	1979	33	1327- 1361	201- 300	33	1327- 1361
400	2731	33	1327- 1361	301- 400	25	2477- 2503
500	3559	33	1327- 1361	401- 500	27	2971- 2999
600	4397	33	1327- 1361	501- 600	29	4297- 4327
700	5273	33	1327- 1361	601- 700	29	4831- 4861
800	6131	33	1327- 1361	701- 800	31	5591- 5623
900	6991	33	1327- 1361	801- 900	29	6491- 6521
1000	7907	33	1327- 1361	901-1000	29	7253- 7283
1100	8821	33	1327- 1361	1001-1100	33	8467- 8501
1200	9721	35	9551- 9587	1101-1200	35	9551- 9587
1300	10651	35	9551- 9587	1201-1300	33	9973-10007
1400	11633	35	9551- 9587	1301-1400	31	10799-10831
1500	12547	35	9551- 9587	1401-1500	33	11743-11777
1600	13487	35	9551- 9587	1501-1600	35	12853-12889
1700	14503	35	9551- 9587	1601-1700	35	14107-14143
1800	15391	35	9551- 9587	1701-1800	29	14983-15013
1900	16369	43	15683-15727	1801-1900	43	15683-15727
2000.	17387	43	15683-15727	1901-2000	33	17257-17291
2100	18311	43	15683-15727	2001-2100	29	17627-17657
2200	19421	43	15683-15727	2101-2200	39	19333-19373
2300	20353	51	19609-19661	2201-2300	51	19609-19661
2400	21379	51	19609-19661	2301-2400	39	20809-20849

4. The "Prime Theorem" Conjecture

Legendre and Gauss in the early nineteenth century conjectured that the number of primes not greater than some large integer would be given by the term $n/\ln n$, where n is the integer and ln n its natural logarithm. The tables presented here compare this conjectured number of primes with the actual number of primes for twenty one groups of one thousand integers, up to the limiting integer of 21,000.

This prime theorem conjecture can also be applied to small groups of large integers. For a group of large integers ranging from n1 to n2, the increment in the number of primes is given as $n2/\ln n2 - n1/\ln n1$. The proportion of prime numbers in this group can so be expressed, closely, as $1/\ln n$, where n is a representative intermediate integer within the group

The conjected number of primes occurring within a specified range of integers is less than the actual number of primes, but otherwise the two agree reasonably well. Both conjected and actual number of primes increase as the limiting integer is increased,

but at decreasing rates. This is shown in the accompanying tables of actual and conjectured primes by the frequency with which they occur within successive groups of one thousand integers. The rate of decrease in the number of conjected primes is based on a monotoni. transendental mathematical relation and is quite regular (except for slight irregularities introduced by rounding). In contrast, the rate of decrease in the actual number of primes, although similar to that for conjected primes, is irregular. The table also shows the conjectured number of primes not greater that the large integer n as given by the term n/ln n, and the proportion of primes 1/ln n for groups of integers that include this integer.

The prime theorem conjecture also states that the product of an integer and its natural logarithm, rounded to the nearest odd number, is a prime. From accompanying sample tables for conjectured primes presented in the format of the table for actual primes, it is evident that the conjectured number of primes is greater than the actual number. Thus none of the first one hundred conjectured primes are

greater than the integer 461, while the first one hundred actual primes include the integer 523.

The tables for conjectured primes include those ending with the digit '5', and these can not be actual primes for they necessarily are divisible by five. They can be termed "false positives". Since the digit '5' constitutes one fifth the odd numbered digits, these false positives constitute, statistically, one fifth the conjected primes. In addition, other false positives, for example 27, 323, and 17487, are indicated. Included also are some false negatives where actual primes are not indicated in the list of conjectured primes. Examples are 43, 457, and 18191.

Some of these false positives and false negatives correspond to "near misses". For example the number 355 is a conjectured prime but obviously is not an actual prime, while neighboring odd numbers 353 and 359 are actual primes but not conjectured primes.

It has been suggested that the prime number theorem might be reworded to something like "large integers are either composite numbers which can be factored into prime numbers, or themselves are prime

numbers which can be represented, closely, by the expression N ln N, where N is a smaller integer and ln its natural logarithm". Thus the large integer of the anecdote concerning Fermat (Introduction) is a composite number that can be factored into two prime numbers, 112,303 and 898,423, and these can be represented, closely, as 112,303 ~ 11,961 ln 11,961, and 898423 ~ 79,913 ln 79,913.

The First One Hundred Conjectured Primes

1	1	3	5	9	11	13	17	19	23
27	29	33	37	41	45	49	53	55	59
63	69	73	77	81	85	89	93	97	103
107	111	115	119	125	129	133	139	143	147
153	157	161	167	171	177	181	185	191	195
201	205	211	215	221	225	231	235	241	245
251	255	261	267	271	277	281	287	293	297
303	307	313	319	323	329	335	339	345	351
355	361	367	373	377	383	389	395	399	405
411	417	421	427	433	439	443	449	455	461

The Twenty-fourth Group of One Hundred Conjectured Primes

17813	17821	17829	17839	17847	17855	17865	17873	17883	17891
17899	17909	17917	17925	17935	17943	17953	17961	17969	17979
17987	17995	18005	18013	18023	18031	18039	18049	18057	18065
18075	18083	18093	18101	18109	18119	18127	18135	18145	18153
18163	18171	18179	18189	18197	18207	18215	18223	18233	18241
18249	18259	18267	18277	18285	18293	18303	18311	18319	18329
18337	18347	18355	18363	18373	18381	18391	18399	18407	18417
18425	18433	18443	18451	18461	18469	18477	18487	18495	18505
18513	18521	18531	18539	18549	18557	18565	18575	18583	18591
18601	18609	18619	18627	18635	18645	18653	18663	18671	18679

Conjectured Number of Primes and Number of Conjectured Primes

Limit integer	Conj. no.of primes	Actual no.of primes	no. of conj. primes	integer range	Conj. no.of primes	Actual no.of primes	no. of conj. primes
1000	145	169	190	1- 1000	145	169	190
2000	263	304	342	1001- 2000	118	135	152
3000	375	431	486	2001- 3000	112	127	143
4000	482	551	621	3001- 4000	107	120	136
5000	587	670	754	4001- 5000	105	119	133
6000	690	784	884	5001- 6000	103	114	130
7000	791	901	1011	6001- 7000	101	117	127
8000	890	1008	1136	7001- 8000	99	107	125
9000	988	1118	1260	8001- 9000	98	110	124
10000	1086	1230	1382	9001-10000	98	112	122
11000	1182	1336	1503	10001-11000	96	106	121
12000	1278	1439	1623	11001-12000	96	103	120
13000	1372	1548	1741	12001-13000	94	109	118
14000	1466	1653	1859	13001-14000	94	105	118
15000	1560	1755	1976	14001-15000	94	102	117
16000	1653	1863	2092	15001-16000	93	108	116
17000	1745	1961	2207	16001-17000	92	98	115
18000	1837	2065	2322	17001-18000	92	104	115
19000	1929	2159	2436	18001-19000	92	94	114
20000	2019	2263	2549	19001-20000	90	104	113
21000	2110	2361	2262	20001-21000	91	98	113

5. Factorization

A composite number can be factored into smaller numbers; a prime number cannot. Thus a method for factorization can not only identify the factors for a composite number, but also those integers which are prime numbers.

A straightforward procedure for factorization consists in trying out as possible divisors all these prime numbers not greater than the square root of a specified integer. This trial-and-error method is relatively satisfactory for smaller integers such as those with no more than four or five digits, but for larger integers the effort can become overwhelming.

The classical method for factorization is that of Fermat, based on the algebraic relation x^2 - y^2 = (x+y)*(x-y). Both trial-and-error and Fermat methods are readily adapted to computer use; however many computers do not have a capability for dealing with very large integers.

The two thousand four hundred prime numbers presented in these tables involve integers up to 21,380; these are readily handled here. For this the trial-and-error method may have an advantage, particularly for factoring composite integers with more than a small number of factors.

The accompanying factorizations illustrate the trial-and-error method. These include the maximum integer of interest here, 21,380, an integer with a relatively large number of factors, integers known to be primes, and the Eucludian integer 300031. Also included is the integer 11961 that from the Prime Theorem Conjecture should represent the Fermat prime number 112,303. It does not, for here the term N ln N, rounded to an odd number, is 112,307. (This might be regarded as a "near miss".) It can be noted that the integer 11961 is a composite rather than a prime number, as

$$27 \times 443 = 11961$$

Factorizations

Integer to be factored: 21380

Integer	Factor	Quotient
21380	2	10690
10690	2	5345
5345	5	1069
1069	1069	1

The integer 21380 can be factored into

2 2 5 1069

Integer to be factored: 15120

Integer	Factor	Quotient
15120	2	7560
7560	2	3780
3780	2	1890
1890	2	945
945	3	315
315	3	105
105	3	35
35	5	7
7	7	1

The integer 15120 can be factored into

2 2 2 2 3 3 3 5 7

Integer to be factored: 21379

Integer	Factor	Quotient
21379	21379	1

The integer 21379 is a prime number

Integer to be factored: 365

Integer	Factor	Quotient
365	5	73
73	73	1

The integer 365 can be factored into

5 73

Integer to be factored: 1728

Integer	Factor	Quotient
1728	2	864
864	2	432
432	2	216
216	2	108
108	2	54
54	2	27
27	3	9
9	3	3
3	3	1

The integer 1728 can be factored into

2 2 2 2 2 2 3 3 3

Integer to be factored: 893

Integer	Factor	Quotient
893	19	47
47	47	1

The integer 893 can be factored into

19 47

Integer to be factored: 11961

Integer	Factor	Quotient
11961	3	3987
3987	3	1329
1329	3	443
443	443	1

The integer 11961 can be factored into

3 3 3 443

Integer to be factored: 19

Integer	Factor	Quotient
19	19	1

The integer 19 is a prime number

Integer to be factored: 30031

Integer	Factor	Quotient
30031	59	509
509	509	1

The integer 30031 can be factored into

59 509

Factorization, (continued)

Integer to be factored: 5280

Integer	Factor	Quotient
5280	2	2640
2640	2	1320
1320	2	660
660	2	330
330	2	165
165	3	55
55	5	11
11	11	1

The integer 5280 can be factored into

2 2 2 2 2 3 5 11

Integer to be factored: 9211

Integer	Factor	Quotient
9211	61	151
151	151	1

The integer 9211 can be factored into

61 151

Integer to be factored: 19321

Integer	Factor	Quotient
19321	139	139
139	139	1

The integer 19321 can be factored into

139 139

6. Inverses and Hamming-Kinney Numbers

The inverse of a number has its same digits but in inverse order. Thus inverse of the integer 12,345 is 54,321, and vice versa. Special cases where the inverse of a number is also the number itself, for example 12,321 and its inverse 12,321, are termed "palindromes". A palindrome that also is a prime, for example 17,471, is a Hamming-Kinney palindrome.

An accompanying table lists the forty seven Hamming-Kinney palindromes included in the 2,400 primes of this report. Five of these primes have only one digit. There are twenty one two-digit primes, only one of which is a Hamming-Kinney palindrome. There are one hundred forty three three-digit primes; only fifteen of these are Hamming-Kinney palindromes. There are no four-digit Hamming-Kinney palindromes. The seven hundred primes with five digits, as listed in this report, include the twenty six Hamming-Kinney palindromes.

Primes whose inverses are also primes are Hamming-Kinney numbers. An example is the prime number 1979, whose inverse 9791 is also a prime. Such pairs of primes form Hamming-Kinney pairs. Examples are 13 and 31, 7219 and 9127, etc. There are four Hamming-Kinney pairs in the twenty one two-digit primes. The nine hundred consecutive three digit integers include one hundred forty

three primes, and thirty of these form fifteen Hamming-Kinney pairs. The nine thousand four digit integers have one thousand thirty prime numbers, including one hundred one Hamming-Kinney pairs. These Hamming-Kinney pairs are listed in an accompanying table, where the smaller integer of the pair is given first.

It has been observed that inverses of some primes are also primes, and that this occurs more often than might be expected. An extreme example is provided by the fourteen prime numbers in the one hundred consecutive integers between seven hundred ano eight hundred. Of these fourteen primes, twelve have inverses that also are primes, as opposed to a perhaps expected total of three or four. Another example is afforded by the one hundred integers from thirty two hundred to thirty three hundred. Here there are eleven prime numbers of which six, or slightly more than half, have inverses that also are prime numbers. This contrasts sharply with the one or two suggested by the prime theorem conjecture of section 4.

The Hamming hypothesis states that there is some rule, analogous perhaps to the rule of three, that describes this unusual situation. (The rule of three states that if the sum of digits in an integer is divisible by three, the integer itself is div.sible by

three). A possible explanation for the unusual situation here is that the final digit of a prime number (when expressed decimally to base ten) must be a 1, 3, 7, or 9. Thus the initial digit of its inverse must also be one of these four selected digits. Thus inverses are bunched into groups with an initial digit that is one of these pre-selected four. This is illustrated in an accompanying table. There the forty five primes between integers 600 through 900, along with their inverses, are listed. Those inverses that also are primes are underlined. Here the bunching effect is quite evident. It so becomes apparent that the sequences of primes in the two examples above, and in similiar situations on which the Hamming hypothesis is based, are not randomly chosen samples, but ones which inadvertantly were especially selected.

Hamming-Kinney Palindromes

(included in the first 2,400 prime numbers)

1	2	3	5	7	11	101
131	151	181	191	313	353	373
383	727	757	787	797	919	929
10301	10501	10601	11311	11411	12421	12721
12821	13331	13831	13931	14341	14741	15451
15551	16061	16361	16561	16661	17471	17971
18181	18481	19391	19891	19991		

Hamming-Kinney Pairs

(included in the first 10,000 integers)

13-31	17-71	37-73	79-97	107-701
113-311	159-941	157-751	167-761	179-971
199-991	337-733	347-743	359-953	389-983
709-907	739-937	769-967	1013-3101	1021-1201
1031-1301	1033-3301	1061-1601	1069-9601	1091-1901
1097-7901	1103-3011	1109-9011	1151-1511	1153-3511
1181-1811	1193-3911	1213-3121	1217-7121	1223-3221
1229-9221	1231-1321	1237-7321	1249-9421	1259-9521
1279-9721	1283-3821	1381-1831	1399-9931	1409-9041
1439-9314	1451-1541	1471-1741	1487-7841	1499-9941
1523-3251	1559-9551	1583-3851	1597-7951	1619-9161
1657-7561	1669-9661	1723-3271	1733-3371	1753-3571
1789-9871	1847-7481	1867-7681	1879-9781	1913-3191
1937-7391	1949-9491	1973-3791	1979-9791	3019-9103
3023-3203	3037-7303	3049-9403	3067-7603	3083-3803
3089-9803	3109-9013	3163-3613	3169-9613	3257-7523
3299-9923	3319-9133	3343-3433	3347-7433	3359-5933
3373-3733	3389-9833	3463-3643	3467-7643	3469-9643
3527-7253	3583-3853	3697-7963	3719-9173	3767-7673
3889-9833	3917-7193	3929-9293	7027-7207	7057-5707
7129-9217	7187-7817	7229-9227	7297-7927	7349-9437
7457-7547	7459-9547	7529-9257	7577-7757	7589-9857
7649-9467	7687-7867	7699-9967	7879-9787	7949-9497
9029-9029	9349-9439	9479-9749	9679-9769	

Inverses of Primes

(in the Integer Range 600-900)

601-106	607-706	613-316	617-716	619-916
631-136	641-146	643-346	647-746	653-356
659-956	661-166	673-376	677-776	683-386
691-916	701-107	709-907	719-917	727-727
733-337	739-937	743-347	751-157	757-757
761-167	769-967	773-377	787-787	797-797
809-908	811-118	821-128	823-328	827-728
829-928	839-938	853-358	857-758	859-958
863-368	877-778	881-188	883-388	887-788

7. Reciprocals

Reciprocals of the prime numbers are repeating decimal fractions (primes 1, 2, and 5 excepted). These repeating fractions show a number of initial zeros (after the decimal point) that equals the number of digits in the prime number minus one. For example, the reciprocal of 7 is

0.142,857,142,857,142,857 . . .

with zero initial zeros (1-1 = 0). Reciprocal of the prime number 271 is

0.003,690,036,900,369,003 . . .

with 3-1 = 2 initial zeros.

The repeating portion of these decimal fractions is the "period integer" and its number of digits the "period length". Period integer for the prime number 7 is 142,857, and the period length is 6, an even number. The (reciprocal of the) prime number 271 has the period integer 36,900 and the period length of 5 digits, an odd number. The accompanying table shows selected sequences of twenty prime numbers with their respective period lengths. (These period lengths are taken from the listing for the first 1,370,471 primes tabulated by Samuel Yates in 1975.)

The maximum number of digits in a period integer is one less than the prime number itself. Thus the maximum period length for the prime number 7 is 6, and that for the prime number 271 is 270. Actual period lengths may be less than maximum, but necessarily are sub-multiples of it. For the prime number 271 the

actual period length is 54, so that the sub-multiple is 270/54 = 5. For the prime number 7 the sub-multiple is 6/6 = 1, indicating a "full period" integer. Statistically, three eighths (closely) of all period integers have full periods. For the smallish number of period integers in the accompanying table, the actual fraction of full period integers is three and one half eighths, in satisfactory agreement with the value for very large numbers of such integers.

Two thirds, or nearly 67 percent, of all period integers are even numbered, with the rest being odd numbered. The actual percentage of even numbered period integers in the accompanying table with eighty items is 69 percent, in substantial agreement with the statistical value pertaining to very large numbers.

Included in the table for reciprocals are the associated sub-multiples, the ratios of maximum to actual values for period lengths. Maximum sub-multiple for the reciprocals of the 2400 primes presented here is 374, given by the ratio 21,318/57 for the 2395th prime number 21,319. Of interest is the near-by pair of prime number twins separated by the integer 21,648. Here one of the twins has a period length of 21,646 digits, the other only 11 digits. Corresponding sub-multiples 1 and 1938 show a rather remarkable difference for two prime numbers that otherwise seem quite similar.

Prime number	Reciprocal Period Lengths actual	maximum	Sub-multiple
3	1	2	2
7	6	6	1
11	2	10	5
13	6	12	2
17	16	16	1
19	18	18	1
23	22	22	1
29	28	28	1
31	15	30	2
37	3	36	12
41	5	40	8
43	21	42	2
47	46	46	1
53	13	52	4
59	58	58	1
61	60	60	1
67	33	66	2
71	35	70	2
73	8	72	9
79	13	78	6
6163	79	6162	78
6173	3086	6172	2
6197	3098	6196	2
6199	3099	6198	2
6203	443	6202	14
6211	6230	6230	1
6217	6216	6216	1
6221	6220	6220	1
6229	2076	6228	3
6247	6246	6246	1
6257	6256	6256	1
6263	6262	6262	1
6269	6268	6268	1
6271	1045	6270	6
6277	1569	6276	4
6287	6286	6286	1
6299	94	6298	67
6301	6300	6300	1
6311	3155	6310	2
6317	3158	6316	2

Prime number	Reciprocal Period Lengths actual	maximum	Sub-multiple
13331	13330	13330	1
13337	13336	13336	1
13339	13338	13338	1
13367	13366	13366	1
13381	13380	13380	1
13397	6698	13396	2
13399	957	13398	14
13411	13410	13410	1
13417	4472	13416	3
13421	13420	13420	1
13441	6720	13440	2
13451	13450	13450	1
13457	13456	13456	1
13463	13462	13462	1
13469	13468	13468	1
13477	6738	13476	2
13487	13486	13486	1
13499	13498	13498	1
13513	4504	13512	2
13523	6761	13522	2
21169	1323	21168	16
21179	21178	21178	1
21187	1177	21186	18
21191	2119	21190	10
21193	7064	21192	3
21211	21210	21210	1
21221	4246	21220	5
21227	10613	21226	2
21247	21246	21246	1
21269	21268	21268	1
21277	1182	21276	18
21283	3547	21282	6
21313	2368	21312	9
21317	10658	21316	2
21319	57	21318	374
21323	10661	21322	2
21341	4268	21340	5
21347	10673	21346	2
21377	21376	21376	1
21379	3054	21378	7

8. FERMAT NUMBERS

The great French mathematician Pierre de Fermat (1601-1665) asserted that numbers of the form $(2^{2^n} + 1)$ are primes, but added that he could not prove it. This assertion is now known as the Fermat Theorem of Binary Powers; such integers are now identified as Fermat numbers. For example the fifth Fermat number, where $n = 5$ and $2^n = 32$, is the ten-digit integer 4,294,967,297.

Then in 1723 the famous Swiss mathematician Leonard Euler showed, perhaps by congruence methods and noting that factors of many Fermat numbers are of the form $(64n + 1)$, that this fifth Fermat number is a composite rather than a prime, and that it can be factored into 641 times 6,700,417.

Fermat numbers can become very large. The ninth Fermat number has 155 digits, and would require about three ordinary typewritten lines for a print-out. In general each successive Fermat number has twice as many digits as its predecessor, so that ordinarily only the first five Fermat numbers can be displayed on an personal computer.

n	2^n	Fermat number	nature
1	2	5	prime
2	4	17	prime
3	8	257	prime
4	16	65,537	prime
5	32	4,294,957,297	composite
6	64	(20 digit integer)	composite
-	-	-	-
9	512	(155 digit integer)	composite

It has been reported that of the first seventy three Fermat numbers at least twelve are composites (numbers 5, 6, 7, 8, 9, 11, 12, 18, 23, 36, 38, and 73). For example, the ninth Fermat number, as indicated in the table, is a composite integer with one hundred fifty five digits. Only recently, and after considerable computer effort, has this been factored into three prime numbers.

9. That Final Digit

The final digit of a multi-digit prime number cannot be a five or a zero, for then it would be a a composite number divisible by five. Nor can this final digit be an even number, for then it would be a composite divisible by two. Thus the final digit for any prime number must be a one, three, seven, or nine (in decimal notation).

The above observation indicates that simple inspection of the final digit of any decimal integer, no matter how large, can in some circumstances identify it as a composite number. This leads to speculation that perhaps there is some method whereby prime numbers can be identified simply and directly.

In search for this method, it first is noted that the initial one hundred consecutive multi-digit primes contain twenty four with one as the final digit, twenty six with the final digit three, twenty five with a seven, and twenty five with a nine. This approximately even distribution agrees with expectation. It also is noted that these four possible final digits constitute only four out of the ten digits. Thus about sixty percent of a large group of consecutive integers are composites, and the remaining forty percent might, or might not, be primes.

This simple inspection method for identifying composites can be augmented by the well-known "rule of three". This states that if the sum of the digits in an integer is divisible by three, the integer itself is divisible by three and hence it is not a prime number. Somewhat similar, but perhaps less well known, is the "rule of eleven". This states that if the algebraic sum obtained by alternately adding and subtracting successive digits of an integer is zero or some multiple of eleven, the integer is divisible by eleven. Hence it is a composite number and is not a prime.

Example. Classify each of the following integers as a composite or a primes:

(a) 1066, (b) 1775, (c) 3351, (d) 9031,

(e) 12,847, (f) 21,319, (g) 40,497.

Answer. The final digit of the integer 1066 is an even number, hence 1066 is divisible by two and is not a prime. The final digit of the integer 1775 is divisible by 5, hence 1775 also is not a prime.

Sum of digits for the third and subsequent integers above is 12, 13, 22, 16, or 24. Of these, the sums 12 and 24 are divisible by three, so that the integers 3351 and 40,947 are not prime numbers.

For the remaining three integers, 9031, 12,887, and 21,319, each has a final digit that might indicate a prime number. Also, none of their digit sums are divisible by three. Hence each of these three integers might be a prime number. But alternating digit sums for these are 11, -4, and 13 respectively; hence the integer 9031 is a multiple of eleven, and so is a composite. Neither of the other two integers, 12,847 and 21,319, can be classified* as either a prime or a composite by these simple methods.

The screening methods above can identify about three quarters (actually 75.7575 . . %) of all decimal integers, no matter how large, as being composites (non-primes). Each of the remaining one quarter integers is individually a possible prime, but it could be a composite.

These screening methods pertain to integers in decimal form. But it can be noted that prime numbers do not depend on the number base; thus the duodecimal (base twelve) prime number 5287 is also the decimal prime number 9031, and the decimal composite number 1775 is also the duodecimal composite number 1 0 3 (11). Sometimes algebraic manipulations are simpler with

*12,847 = 29×443; 21,318 is a prime number.

numbers to base twelve than with the corresponding numbers to base ten. In the search for prime numbers, it can be noted that only those duodecimal numbers with a final digit of 1, 5, 7, or 11, (four out of twelve) can be primes. Hence here the final digit can indentify two thirds of all duodecimal integers as composites, versus only four out of ten for decimal numbers. Conversion of a decimal number to duodecimal thus might offer some simplification for identification of large prime numbers.

To examine the final digit of a duodecimal integer, conversion of an entire decimal integer is not necessary. Simple divisior of the decimal integer by twelve and examination of the remainder suffices. For example, the decimal integer 2151 when divided by twelve gives the quotient 179, plus a remainder of 3/12. The final duodecimal digit thus is 3, so that this decimal integer 2151 is not a prime number. (The "modulo" command MOD, available on many computers, provides such remainders; thus 2151 MOD (12) = 3).

In this connection it can be noted that the duodecimal integer 1 2 (11) 3 has a final digit that indicates a composite number, while the final digit of the decimal equivalent 2151 indicates a possible prime. (However, sum of the decimal digits is nine, indicating

that 2151 actually is a composite.) Then the decimal number 1775 has a final digit that shows a composite, whereas in duodecimal form, 1 0 3 (11), the final digit indicates a possible prime. It so seems that duodecimal notation might offer advantage in some situations, while decimal notations may be advantageous in others. Thus it appears that there is no real benefit offered by duodecimal notation in the search for very large prime numbers.

In conclusion, as of now there seems to be no simple method for identifying integers which actually are primes. But as a final thought it can be observed that advances in number theory continue, so that in the future

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